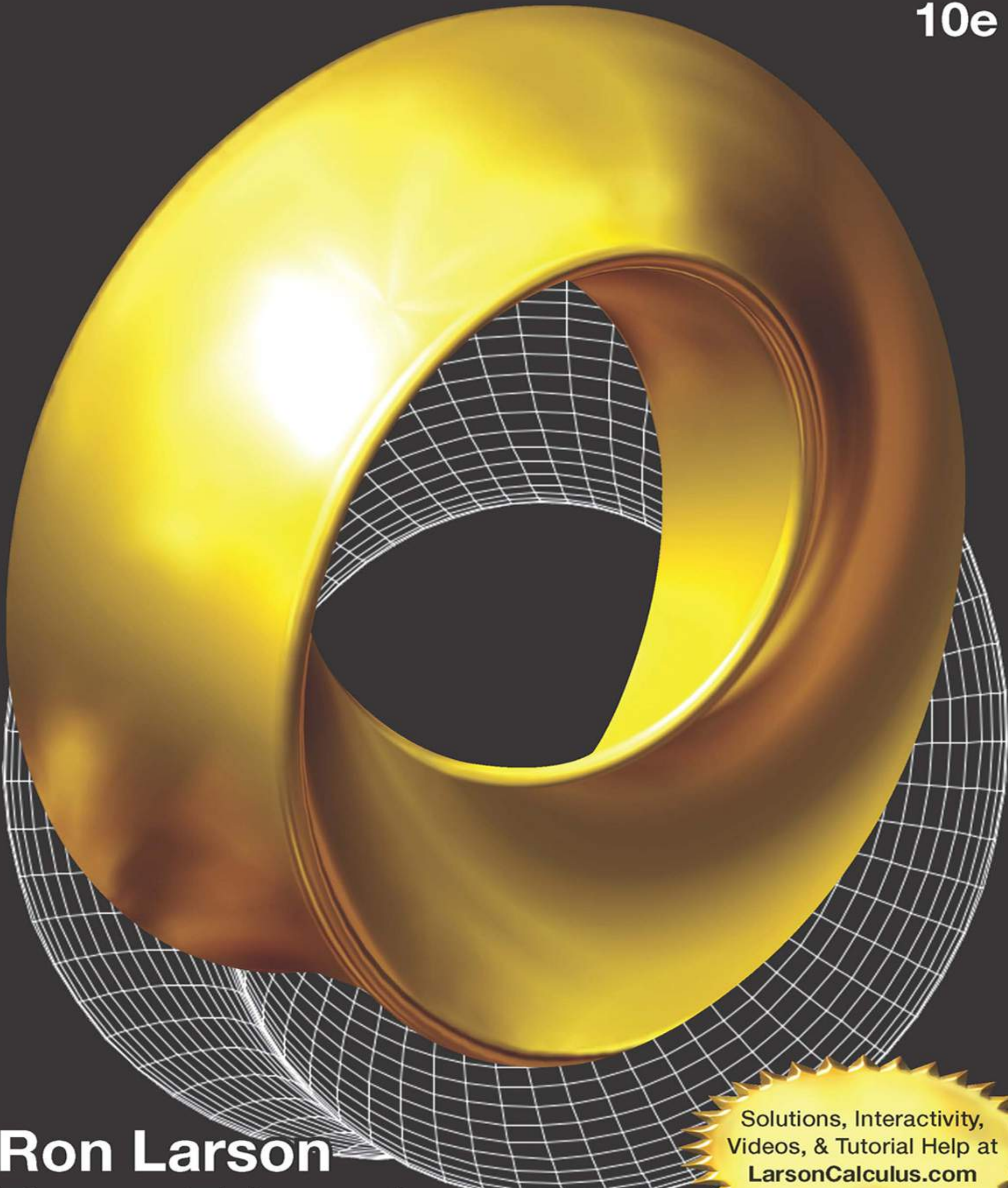
CALCULUS

10e

$$x = (\sin u)\left[7 + \cos\left(\frac{u}{3} - 2v\right) + 2\cos\left(\frac{u}{3} + v\right)\right] \quad y = (\cos u)\left[7 + \cos\left(\frac{u}{3} - 2v\right) + 2\cos\left(\frac{u}{3} + v\right)\right] \quad z = \sin\left(\frac{u}{3} - 2v\right) + 2\sin\left(\frac{u}{3} + v\right)$$



Solutions, Interactivity, Videos, & Tutorial Help at LarsonCalculus.com

Ron Larson
Bruce Edwards

Index of Applications

Engineering and Physical Sciences

(continued on back inside cover)

Tear out Formula Cards for Homework Success.

DERIVATIVES AND INTEGRALS

Basic Differentiation Rules

1. $\frac{d}{dx}[cu] = cu'$
2. $\frac{d}{dx}[u \pm v] = u' \pm v'$
3. $\frac{d}{dx}[uv] = uv' + vu'$
4. $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$
5. $\frac{d}{dx}[c] = 0$
6. $\frac{d}{dx}[u^n] = nu^{n-1}u'$
7. $\frac{d}{dx}[x] = 1$
8. $\frac{d}{dx}[|u|] = \frac{u}{|u|}(u'), \quad u \neq 0$
9. $\frac{d}{dx}[\ln u] = \frac{u'}{u}$
10. $\frac{d}{dx}[e^u] = e^u u'$
11. $\frac{d}{dx}[\log_a u] = \frac{u'}{(\ln a)u}$
12. $\frac{d}{dx}[a^u] = (\ln a)a^u u'$
13. $\frac{d}{dx}[\sin u] = (\cos u)u'$
14. $\frac{d}{dx}[\cos u] = -(\sin u)u'$
15. $\frac{d}{dx}[\tan u] = (\sec^2 u)u'$
16. $\frac{d}{dx}[\cot u] = -(\csc^2 u)u'$
17. $\frac{d}{dx}[\sec u] = (\sec u \tan u)u'$
18. $\frac{d}{dx}[\csc u] = -(\csc u \cot u)u'$
19. $\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1 - u^2}}$
20. $\frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1 - u^2}}$
21. $\frac{d}{dx}[\arctan u] = \frac{u'}{1 + u^2}$
22. $\frac{d}{dx}[\operatorname{arccot} u] = \frac{-u'}{1 + u^2}$
23. $\frac{d}{dx}[\operatorname{arcsec} u] = \frac{u'}{|u|\sqrt{u^2 - 1}}$
24. $\frac{d}{dx}[\operatorname{arccsc} u] = \frac{-u'}{|u|\sqrt{u^2 - 1}}$
25. $\frac{d}{dx}[\sinh u] = (\cosh u)u'$
26. $\frac{d}{dx}[\cosh u] = (\sinh u)u'$
27. $\frac{d}{dx}[\tanh u] = (\operatorname{sech}^2 u)u'$
28. $\frac{d}{dx}[\coth u] = -(\operatorname{csch}^2 u)u'$
29. $\frac{d}{dx}[\operatorname{sech} u] = -(\operatorname{sech} u \tanh u)u'$
30. $\frac{d}{dx}[\operatorname{csch} u] = -(\operatorname{csch} u \coth u)u'$
31. $\frac{d}{dx}[\sinh^{-1} u] = \frac{u'}{\sqrt{u^2 + 1}}$
32. $\frac{d}{dx}[\cosh^{-1} u] = \frac{u'}{\sqrt{u^2 - 1}}$
33. $\frac{d}{dx}[\tanh^{-1} u] = \frac{u'}{1 - u^2}$
34. $\frac{d}{dx}[\coth^{-1} u] = \frac{u'}{1 - u^2}$
35. $\frac{d}{dx}[\operatorname{sech}^{-1} u] = \frac{-u'}{u\sqrt{1 - u^2}}$
36. $\frac{d}{dx}[\operatorname{csch}^{-1} u] = \frac{-u'}{|u|\sqrt{1 + u^2}}$

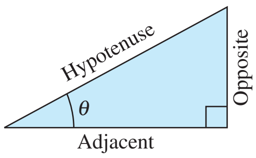
Basic Integration Formulas

1. $\int kf(u)\,du = k\int f(u)\,du$
2. $\int [f(u) \pm g(u)]\,du = \int f(u)\,du \pm \int g(u)\,du$
3. $\int du = u + C$
4. $\int u^n\,du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$
5. $\int \frac{du}{u} = \ln|u| + C$
6. $\int e^u\,du = e^u + C$
7. $\int a^u\,du = \left(\frac{1}{\ln a}\right)a^u + C$
8. $\int \sin u\,du = -\cos u + C$
9. $\int \cos u\,du = \sin u + C$
10. $\int \tan u\,du = -\ln|\cos u| + C$
11. $\int \cot u\,du = \ln|\sin u| + C$
12. $\int \sec u\,du = \ln|\sec u + \tan u| + C$
13. $\int \csc u\,du = -\ln|\csc u + \cot u| + C$
14. $\int \sec^2 u\,du = \tan u + C$
15. $\int \csc^2 u\,du = -\cot u + C$
16. $\int \sec u \tan u\,du = \sec u + C$
17. $\int \csc u \cot u\,du = -\csc u + C$
18. $\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + C$
19. $\int \frac{du}{a^2 + u^2} = \frac{1}{a}\arctan \frac{u}{a} + C$
20. $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\operatorname{arcsec} \frac{|u|}{a} + C$

TRIGONOMETRY

Definition of the Six Trigonometric Functions

Right triangle definitions, where $0 < \theta < \pi/2$.

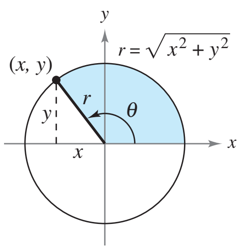


$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

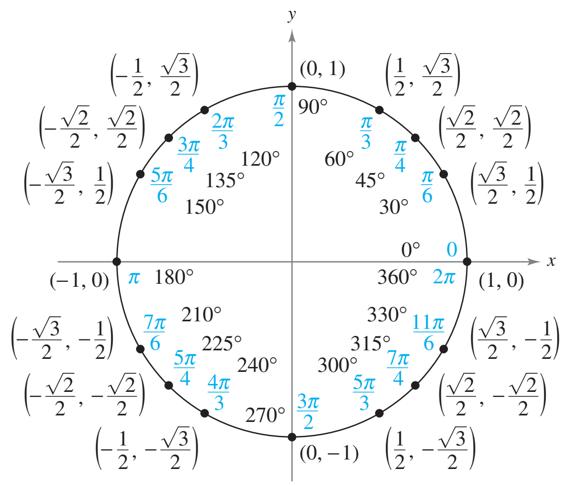
Circular function definitions, where θ *is any angle.*



$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y}$$

$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$



Reciprocal Identities

$$\sin x = \frac{1}{\csc x} \qquad \sec x = \frac{1}{\cos x} \qquad \tan x = \frac{1}{\cot x}$$

$$\csc x = \frac{1}{\sin x} \qquad \cos x = \frac{1}{\sec x} \qquad \cot x = \frac{1}{\tan x}$$

Quotient Identities

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x \qquad 1 + \cot^2 x = \csc^2 x$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x \qquad \cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\csc\left(\frac{\pi}{2} - x\right) = \sec x \qquad \tan\left(\frac{\pi}{2} - x\right) = \cot x$$

$$\sec\left(\frac{\pi}{2} - x\right) = \csc x \qquad \cot\left(\frac{\pi}{2} - x\right) = \tan x$$

Even/Odd Identities

$$\sin(-x) = -\sin x \qquad \cos(-x) = \cos x$$

$$\csc(-x) = -\csc x \qquad \tan(-x) = -\tan x$$

$$\sec(-x) = \sec x \qquad \cot(-x) = -\cot x$$

Sum and Difference Formulas

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

Double-Angle Formulas

$$\sin 2u = 2 \sin u \cos u$$

$$\cos 2u = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$$

$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u}$$

Power-Reducing Formulas

$$\sin^2 u = \frac{1 - \cos 2u}{2}$$

$$\cos^2 u = \frac{1 + \cos 2u}{2}$$

$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

Sum-to-Product Formulas

$$\sin u + \sin v = 2\sin\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\sin u - \sin v = 2\cos\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

$$\cos u + \cos v = 2\cos\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\cos u - \cos v = -2\sin\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

Product-to-Sum Formulas

$$\sin u \sin v = \frac{1}{2}[\cos(u - v) - \cos(u + v)]$$

$$\cos u \cos v = \frac{1}{2}[\cos(u - v) + \cos(u + v)]$$

$$\sin u \cos v = \frac{1}{2}[\sin(u + v) + \sin(u - v)]$$

$$\cos u \sin v = \frac{1}{2}[\sin(u + v) - \sin(u - v)]$$

Calculus

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Calculus

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Contents

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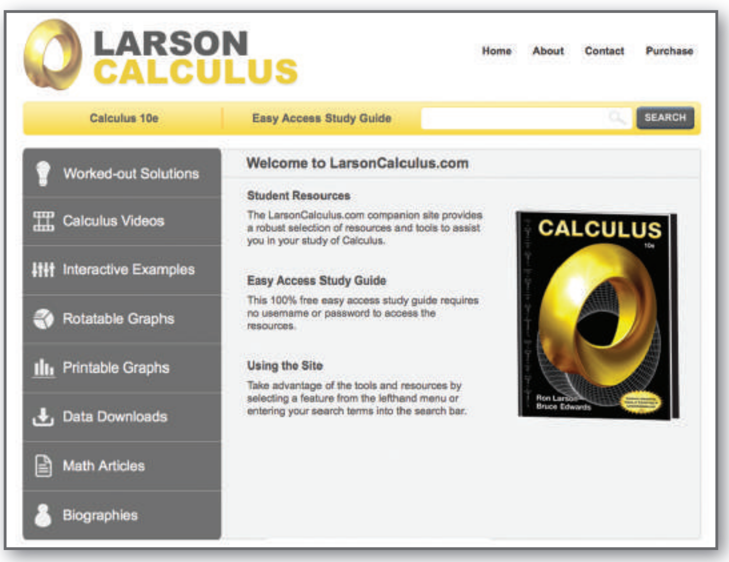
Preface

Welcome to *Calculus*, Tenth Edition. We are proud to present this new edition to you. As with all editions, we have been able to incorporate many useful comments from you, our user. For this edition, we have introduced some new features and revised others. You will still find what you expect – a pedagogically sound, mathematically precise, and comprehensive textbook.

We are pleased and excited to offer you something brand new with this edition – a companion website at LarsonCalculus.com. This site offers many resources that will help you as you study calculus. All of these resources are just a click away.

Our goal for every edition of this textbook is to provide you with the tools you need to master calculus. We hope that you find the changes in this edition, together with **LarsonCalculus.com,** will accomplish just that.

In each exercise set, be sure to notice the reference to **CalcChat.com.** At this free site, you can download a step-by-step solution to any odd-numbered exercise. Also, you can talk to a tutor, free of charge, during the hours posted at the site. Over the years, thousands of students have visited the site for help. We use all of this information to help guide each revision of the exercises and solutions.



New To This Edition

NEW LarsonCalculus.com

This companion website offers multiple tools and resources to supplement your learning. Access to these features is free. Watch videos explaining concepts or proofs from the book, explore examples, view three-dimensional graphs, download articles from math journals and much more.

NEW Chapter Opener

Each Chapter Opener highlights real-life applications used in the examples and exercises.

NEW Interactive Examples

Examples throughout the book are accompanied by Interactive Examples at LarsonCalculus.com. These interactive examples use Wolfram's free CDF Player and allow you to explore calculus by manipulating functions or graphs, and observing the results.

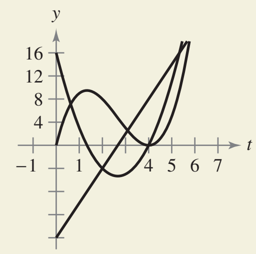
NEW Proof Videos

Watch videos of co-author Bruce Edwards as he explains the proofs of theorems in *Calculus*, Tenth Edition at LarsonCalculus.com.

NEW How Do You See It?

The How Do You See It? feature in each section presents a real-life problem that you will solve by visual inspection using the concepts learned in the lesson. This exercise is excellent for classroom discussion or test preparation.

118. HOW DO YOU SEE IT? The figure shows the graphs of the position, velocity, and acceleration functions of a particle.



(a) Copy the graphs of the functions shown. Identify each graph. Explain your reasoning. To print an enlarged copy of the graph, go to *MathGraphs.com*.

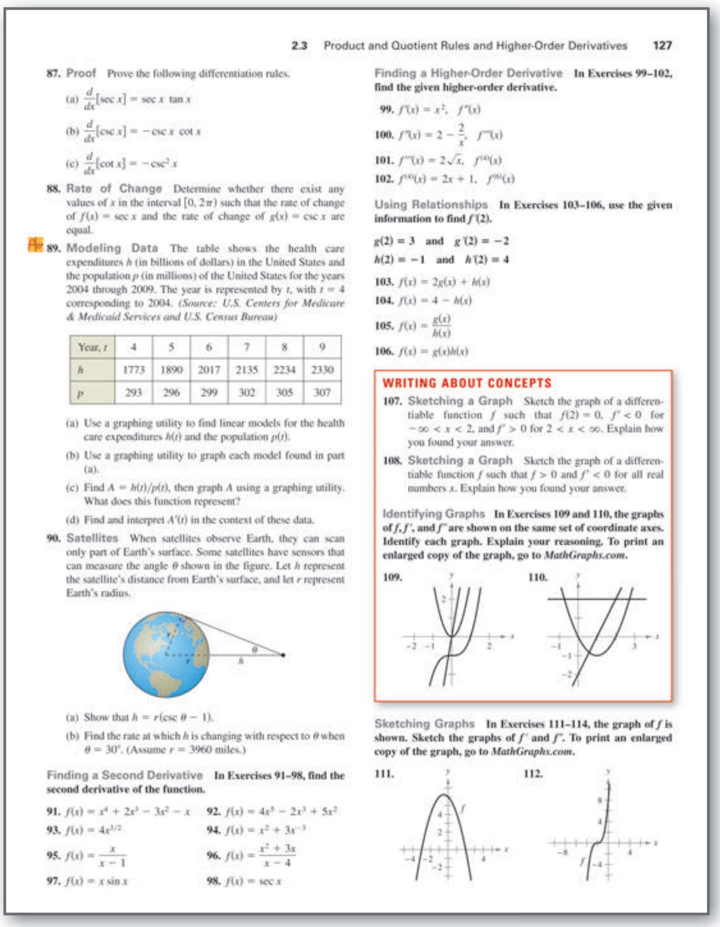
(b) On your sketch, identify when the particle speeds up and when it slows down. Explain your reasoning.

REVISED Remark

These hints and tips reinforce or expand upon concepts, help you learn how to study mathematics, caution you about common errors, address special cases, or show alternative or additional steps to a solution of an example.

REVISED Exercise Sets

The exercise sets have been carefully and extensively examined to ensure they are rigorous and relevant and include all topics our users have suggested. The exercises have been reorganized and titled so you can better see the connections between examples and exercises. Multi-step, real-life exercises reinforce problem-solving skills and mastery of concepts by giving students the opportunity to apply the concepts in real-life situations.



2.3 Product and Quotient Rules and Higher-Order Derivatives 127

87. Proof Prove the following differentiation rules.

(a) $\frac{d}{dx}[\sec x] = \sec x \tan x$

(b) $\frac{d}{dx}[\csc x] = -\csc x \cot x$

(c) $\frac{d}{dx}[\cot x] = -\csc^2 x$

88. Rate of Change Determine whether there exist any values of x in the interval $[0, 2\pi)$ such that the rate of change of $f(x) = \sec x$ and the rate of change of $g(x) = \csc x$ are equal.

89. Modeling Data The table shows the health care expenditures h (in billions of dollars) in the United States and the population p (in millions) of the United States for the years 2004 through 2009. The year is represented by t, with $t = 4$ corresponding to 2004. *(Source: U.S. Centers for Medicare & Medicaid Services and U.S. Census Bureau)*

Year, t	4	5	6	7	8	9
h	1773	1890	2017	2135	2234	2330
p	293	296	299	302	305	307

(a) Use a graphing utility to find linear models for the health care expenditures $h(t)$ and the population $p(t)$.

(b) Use a graphing utility to graph each model found in part (a).

(c) Find $A = h(t)/p(t)$, then graph A using a graphing utility. What does this function represent?

(d) Find and interpret $A'(t)$ in the context of these data.

90. Satellites When satellites observe Earth, they can scan only part of Earth's surface. Some satellites have sensors that can measure the angle θ shown in the figure. Let h represent the satellite's distance from Earth's surface, and let r represent Earth's radius.

(a) Show that $h = r(\csc \theta - 1)$.

(b) Find the rate at which h is changing with respect to θ when $\theta = 30°$. (Assume $r = 3960$ miles.)

Finding a Second Derivative In Exercises 91–98, find the second derivative of the function.

91. $f(x) = x^4 + 2x^3 - 3x^2 - x$
92. $f(x) = 4x^5 - 2x^3 + 5x^2$
93. $f(x) = 4x^{3/2}$
94. $f(x) = x^2 + 3x^{-3}$
95. $f(x) = \frac{x}{x-1}$
96. $f(x) = \frac{x^2 + 3x}{x - 4}$
97. $f(x) = x \sin x$
98. $f(x) = \sec x$

Finding a Higher-Order Derivative In Exercises 99–102, find the given higher-order derivative.

99. $f'(x) = x^2, \quad f''(x)$
100. $f''(x) = 2 - \frac{2}{x}, \quad f'''(x)$
101. $f'''(x) = 2\sqrt{x}, \quad f^{(4)}(x)$
102. $f^{(4)}(x) = 2x + 1, \quad f^{(6)}(x)$

Using Relationships In Exercises 103–106, use the given information to find $f'(2)$.

$g(2) = 3$ and $g'(2) = -2$
$h(2) = -1$ and $h'(2) = 4$

103. $f(x) = 2g(x) + h(x)$
104. $f(x) = 4 - h(x)$
105. $f(x) = \frac{g(x)}{h(x)}$
106. $f(x) = g(x)h(x)$

WRITING ABOUT CONCEPTS

107. Sketching a Graph Sketch the graph of a differentiable function f such that $f(2) = 0$, $f' < 0$ for $-\infty < x < 2$, and $f' > 0$ for $2 < x < \infty$. Explain how you found your answer.

108. Sketching a Graph Sketch the graph of a differentiable function f such that $f > 0$ and $f' < 0$ for all real numbers x. Explain how you found your answer.

Identifying Graphs In Exercises 109 and 110, the graphs of f, f', and f'' are shown on the same set of coordinate axes. Identify each graph. Explain your reasoning. To print an enlarged copy of the graph, go to *MathGraphs.com*.

109.

110.

Sketching Graphs In Exercises 111–114, the graph of f is shown. Sketch the graphs of f' and f''. To print an enlarged copy of the graph, go to *MathGraphs.com*.

111.

112.

Table of Content Changes

Appendix A (Proofs of Selected Theorems) now appears in video format at LarsonCalculus.com. The proofs also appear in text form at CengageBrain.com.

Trusted Features

Applications

Carefully chosen applied exercises and examples are included throughout to address the question, "When will I use this?" These applications are pulled from diverse sources, such as current events, world data, industry trends, and more, and relate to a wide range of interests. Understanding where calculus is (or can be) used promotes fuller understanding of the material.

Writing about Concepts

Writing exercises at the end of each section are designed to test your understanding of basic concepts in each section, encouraging you to verbalize and write answers and promote technical communication skills that will be invaluable in your future careers.

Theorems

Theorems provide the conceptual framework for calculus. Theorems are clearly stated and separated from the rest of the text by boxes for quick visual reference. Key proofs often follow the theorem and can be found at LarsonCalculus.com.

Definition of Definite Integral

If f is defined on the closed interval $[a, b]$ and the limit of Riemann sums over partitions Δ

$$\lim_{\|\Delta\|\to 0} \sum_{i=1}^{n} f(c_i)\,\Delta x_i$$

exists (as described above), then f is said to be **integrable** on $[a, b]$ and the limit is denoted by

$$\lim_{\|\Delta\|\to 0} \sum_{i=1}^{n} f(c_i)\,\Delta x_i = \int_a^b f(x)\,dx.$$

The limit is called the **definite integral** of f from a to b. The number a is the **lower limit** of integration, and the number b is the **upper limit** of integration.

Definitions

As with theorems, definitions are clearly stated using precise, formal wording and are separated from the text by boxes for quick visual reference.

Explorations

Explorations provide unique challenges to study concepts that have not yet been formally covered in the text. They allow you to learn by discovery and introduce topics related to ones presently being studied. Exploring topics in this way encourages you to think outside the box.

Historical Notes and Biographies

Historical Notes provide you with background information on the foundations of calculus. The Biographies introduce you to the people who created and contributed to calculus.

Technology

Throughout the book, technology boxes show you how to use technology to solve problems and explore concepts of calculus. These tips also point out some pitfalls of using technology.

Section Projects

Projects appear in selected sections and encourage you to explore applications related to the topics you are studying. They provide an interesting and engaging way for you and other students to work and investigate ideas collaboratively.

Putnam Exam Challenges

Putnam Exam questions appear in selected sections. These actual Putnam Exam questions will challenge you and push the limits of your understanding of calculus.

SECTION PROJECT

St. Louis Arch

The Gateway Arch in St. Louis, Missouri, was constructed using the hyperbolic cosine function. The equation used for construction was

$$y = 693.8597 - 68.7672 \cosh 0.0100333x,$$
$$-299.2239 \le x \le 299.2239$$

where x and y are measured in feet. Cross sections of the arch are equilateral triangles, and (x, y) traces the path of the centers of mass of the cross-sectional triangles. For each value of x, the area of the cross-sectional triangle is

$$A = 125.1406 \cosh 0.0100333x.$$

(Source: Owner's Manual for the Gateway Arch, Saint Louis, MO, by William Thayer)

(a) How high above the ground is the center of the highest triangle? (At ground level, $y = 0$.)

(b) What is the height of the arch? (*Hint:* For an equilateral triangle, $A = \sqrt{3}c^2$, where c is one-half the base of the triangle, and the center of mass of the triangle is located at two-thirds the height of the triangle.)

(c) How wide is the arch at ground level?

Additional Resources

Student Resources

- **Student Solutions Manual for Calculus of a Single Variable**
  (Chapters P–10 of *Calculus*): ISBN 1-285-08571-X

  Student Solutions Manual for Multivariable Calculus
  (Chapters 11–16 of *Calculus*): ISBN 1-285-08575-2

  These manuals contain worked-out solutions for all odd-numbered exercises.

  Enhanced WebAssign *www.webassign.net*
  Printed Access Card: ISBN 0-538-73807-3
  Online Access Code: ISBN 1-285-18421-1
  Enhanced WebAssign is designed for you to do your homework online. This proven and reliable system uses pedagogy and content found in this text, and then enhances it to help you learn calculus more effectively. Automatically graded homework allows you to focus on your learning and get interactive study assistance outside of class. Enhanced WebAssign for *Calculus*, 10e contains the Cengage YouBook, an interactive eBook that contains video clips, highlighting and note-taking features, and more!

  CourseMate
  CourseMate is a perfect study tool for bringing concepts to life with interactive learning, study, and exam preparation tools that support the printed textbook. CourseMate includes: an interactive eBook, videos, quizzes, flashcards, and more!

- **CengageBrain.com—**To access additional materials including CourseMate, visit www.cengagebrain.com. At the CengageBrain.com home page, search for the ISBN of your title (from the back cover of your book) using the search box at the top of the page. This will take you to the product page where these resources can be found.

Instructor Resources

Enhanced WebAssign *www.webassign.net*
Exclusively from Cengage Learning, Enhanced WebAssign offers an extensive online program for *Calculus*, 10e to encourage the practice that is so critical for concept mastery. The meticulously crafted pedagogy and exercises in our proven texts become even more effective in Enhanced WebAssign, supplemented by multimedia tutorial support and immediate feedback as students complete their assignments. Key features include:

- Thousands of homework problems that match your textbook's end-of-section exercises
- Opportunities for students to review prerequisite skills and content both at the start of the course and at the beginning of each section
- Read It eBook pages, Watch It Videos, Master It tutorials, and Chat About It links
- A customizable Cengage YouBook with highlighting, note-taking, and search features, as well as links to multimedia resources
- Personal Study Plans (based on diagnostic quizzing) that identify chapter topics that students will need to master
- A WebAssign Answer Evaluator that recognizes and accepts equivalent mathematical responses in the same way you grade assignments
- A *Show My Work* feature that gives you the option of seeing students' detailed solutions
- Lecture videos, and more!

- **Cengage Customizable YouBook—**YouBook is an eBook that is both interactive and customizable! Containing all the content from *Calculus*, 10e, YouBook features a text edit tool that allows you to modify the textbook narrative as needed. With YouBook, you can quickly re-order entire sections and chapters or hide any content you don't teach to create an eBook that perfectly matches your syllabus. You can further customize the text by adding instructor-created or YouTube video links. Additional media assets include: video clips, highlighting and note-taking features, and more! YouBook is available within Enhanced WebAssign.

- **Complete Solutions Manual for Calculus of a Single Variable, Volume 1**
  (Chapters P–6 of *Calculus*): ISBN 1-285-08576-0

  Complete Solutions Manual for Calculus of a Single Variable, Volume 2
  (Chapters 7–10 of *Calculus*): ISBN 1-285-08577-9

  Complete Solutions Manual for Multivariable Calculus
  (Chapters 11–16 of *Calculus*): ISBN 1-285-08580-9

  The *Complete Solutions Manuals* contain worked-out solutions to all exercises in the text.

- **Solution Builder** (www.cengage.com/solutionbuilder)— This online instructor database offers complete worked-out solutions to all exercises in the text, allowing you to create customized, secure solutions printouts (in PDF format) matched exactly to the problems you assign in class.

- **PowerLecture** (ISBN 1-285-08583-3)—This comprehensive instructor DVD includes resources such as an electronic version of the Instructor's Resource Guide, complete pre-built PowerPoint® lectures, all art from the text in both jpeg and PowerPoint formats, ExamView® algorithmic computerized testing software, JoinIn™ content for audience response systems (clickers), and a link to Solution Builder.

- **ExamView Computerized Testing—** Create, deliver, and customize tests in print and online formats with ExamView®, an easy-to-use assessment and tutorial software. ExamView for *Calculus*, 10e contains hundreds of algorithmic multiple-choice and short answer test items. ExamView® is available on the PowerLecture DVD.

- **Instructor's Resource Guide** (ISBN 1-285-09074-8)—This robust manual contains an abundance of resources keyed to the textbook by chapter and section, including chapter summaries and teaching strategies. An electronic version of the Instructor's Resource Guide is available on the PowerLecture DVD.

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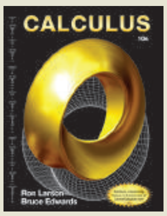
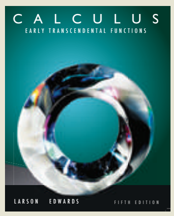
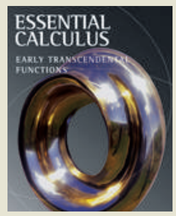
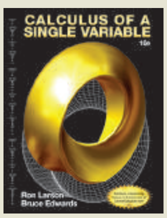
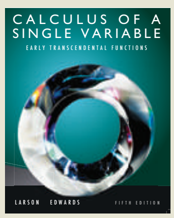
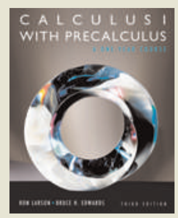
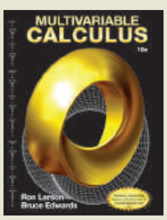
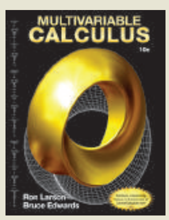
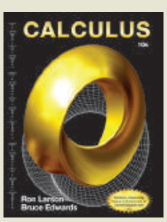
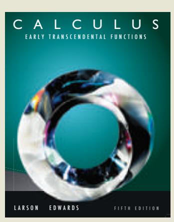
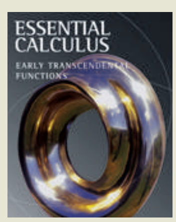
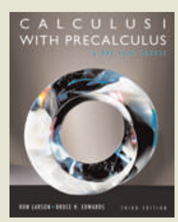
3-semester	Calculus 10e 	Calculus Early Transcendental Functions 5e 	Essential Calculus 	
Single Variable Only	Calculus 10e Single Variable 	Calculus: Early Transcendental Functions 5e Single Variable 		Calculus I with Precalculus 3e 
Multivariable	Calculus 10e Multivariable 	Calculus 10e Multivariable 		
Custom All of these textbook choices can be customized to fit the individual needs of your course.	Calculus 10e 	Calculus: Early Transcendental Functions 5e 	Essential Calculus 	Calculus I with Precalculus 3e 

Calculus

10e

P Preparation for Calculus

Hours of Daylight
(Example 3, p. 33)

Automobile Aerodynamics *(Exercise 96, p. 30)*

Conveyor Design *(Exercise 23, p. 16)*

Cell Phone Subscribers
(Exercise 68, p. 9)

Modeling Carbon Dioxide Concentration *(Example 6, p. 7)*

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P.1 Graphs and Models

- **Sketch the graph of an equation.**
- **Find the intercepts of a graph.**
- **Test a graph for symmetry with respect to an axis and the origin.**
- **Find the points of intersection of two graphs.**
- **Interpret mathematical models for real-life data.**

RENÉ DESCARTES (1596–1650)

Descartes made many contributions to philosophy, science, and mathematics. The idea of representing points in the plane by pairs of real numbers and representing curves in the plane by equations was described by Descartes in his book *La Géométrie*, published in 1637.

See LarsonCalculus.com to read more of this biography.

The Graph of an Equation

In 1637, the French mathematician René Descartes revolutionized the study of mathematics by combining its two major fields—algebra and geometry. With Descartes's coordinate plane, geometric concepts could be formulated analytically and algebraic concepts could be viewed graphically. The power of this approach was such that within a century of its introduction, much of calculus had been developed.

The same approach can be followed in your study of calculus. That is, by viewing calculus from multiple perspectives—*graphically*, *analytically*, and *numerically*—you will increase your understanding of core concepts.

Consider the equation $3x + y = 7$. The point $(2, 1)$ is a **solution point** of the equation because the equation is satisfied (is true) when 2 is substituted for x and 1 is substituted for y. This equation has many other solutions, such as $(1, 4)$ and $(0, 7)$. To find other solutions systematically, solve the original equation for y.

$$y = 7 - 3x \qquad \text{Analytic approach}$$

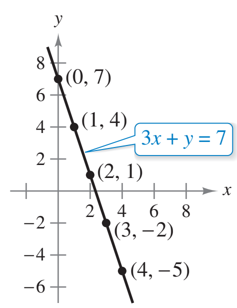
Then construct a **table of values** by substituting several values of x.

x	0	1	2	3	4
y	7	4	1	-2	-5

Numerical approach

From the table, you can see that $(0, 7)$, $(1, 4)$, $(2, 1)$, $(3, -2)$, and $(4, -5)$ are solutions of the original equation $3x + y = 7$. Like many equations, this equation has an infinite number of solutions. The set of all solution points is the **graph** of the equation, as shown in Figure P.1. Note that the sketch shown in Figure P.1 is referred to as the graph of $3x + y = 7$, even though it really represents only a *portion* of the graph. The entire graph would extend beyond the page.

In this course, you will study many sketching techniques. The simplest is point plotting—that is, you plot points until the basic shape of the graph seems apparent.

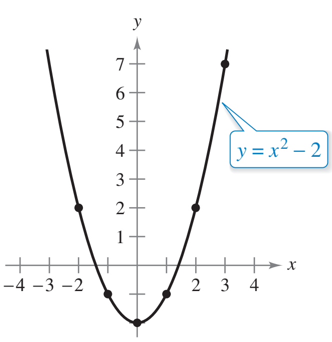


Graphical approach: $3x + y = 7$
Figure P.1

EXAMPLE 1 Sketching a Graph by Point Plotting

To sketch the graph of $y = x^2 - 2$, first construct a table of values. Next, plot the points shown in the table. Then connect the points with a smooth curve, as shown in Figure P.2. This graph is a **parabola.** It is one of the conics you will study in Chapter 10.

x	-2	-1	0	1	2	3
y	2	-1	-2	-1	2	7



The parabola $y = x^2 - 2$
Figure P.2

The Granger Collection, New York

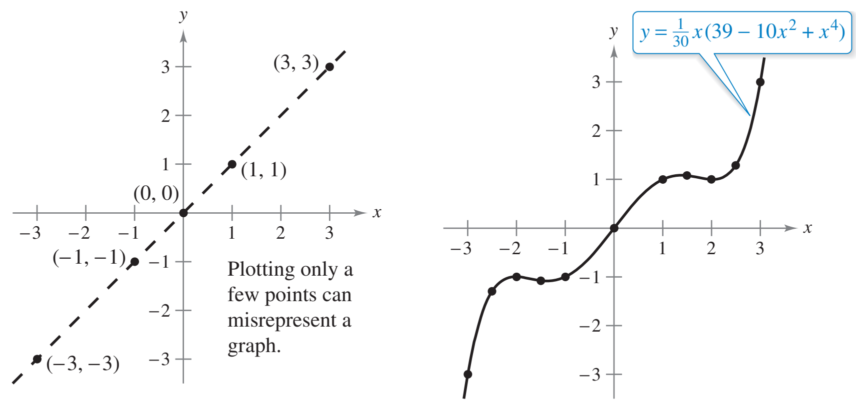
One disadvantage of point plotting is that to get a good idea about the shape of a graph, you may need to plot many points. With only a few points, you could badly misrepresent the graph. For instance, to sketch the graph of

$$y = \frac{1}{30}x(39 - 10x^2 + x^4)$$

you plot five points:

$$(-3, -3), \quad (-1, -1), \quad (0, 0), \quad (1, 1), \quad \text{and} \quad (3, 3)$$

as shown in Figure P.3(a). From these five points, you might conclude that the graph is a line. This, however, is not correct. By plotting several more points, you can see that the graph is more complicated, as shown in Figure P.3(b).



(a) **(b)**

Figure P.3

Exploration

Comparing Graphical and Analytic Approaches Use a graphing utility to graph each equation. In each case, find a viewing window that shows the important characteristics of the graph.

a. $y = x^3 - 3x^2 + 2x + 5$

b. $y = x^3 - 3x^2 + 2x + 25$

c. $y = -x^3 - 3x^2 + 20x + 5$

d. $y = 3x^3 - 40x^2 + 50x - 45$

e. $y = -(x + 12)^3$

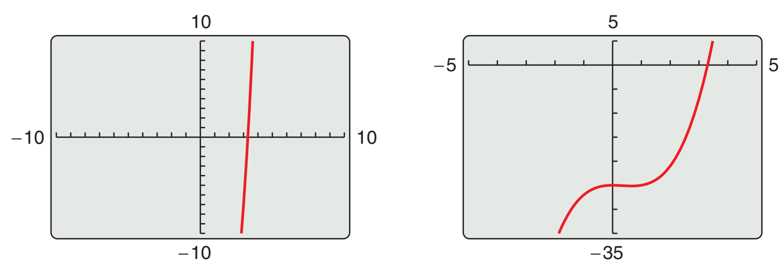
f. $y = (x - 2)(x - 4)(x - 6)$

A purely graphical approach to this problem would involve a simple "guess, check, and revise" strategy. What types of things do you think an analytic approach might involve? For instance, does the graph have symmetry? Does the graph have turns? If so, where are they? As you proceed through Chapters 1, 2, and 3 of this text, you will study many new analytic tools that will help you analyze graphs of equations such as these.

TECHNOLOGY Graphing an equation has been made easier by technology. Even with technology, however, it is possible to misrepresent a graph badly. For instance, each of the graphing utility* screens in Figure P.4 shows a portion of the graph of

$$y = x^3 - x^2 - 25.$$

From the screen on the left, you might assume that the graph is a line. From the screen on the right, however, you can see that the graph is not a line. So, whether you are sketching a graph by hand or using a graphing utility, you must realize that different "viewing windows" can produce very different views of a graph. In choosing a viewing window, your goal is to show a view of the graph that fits well in the context of the problem.



Graphing utility screens of $y = x^3 - x^2 - 25$

Figure P.4

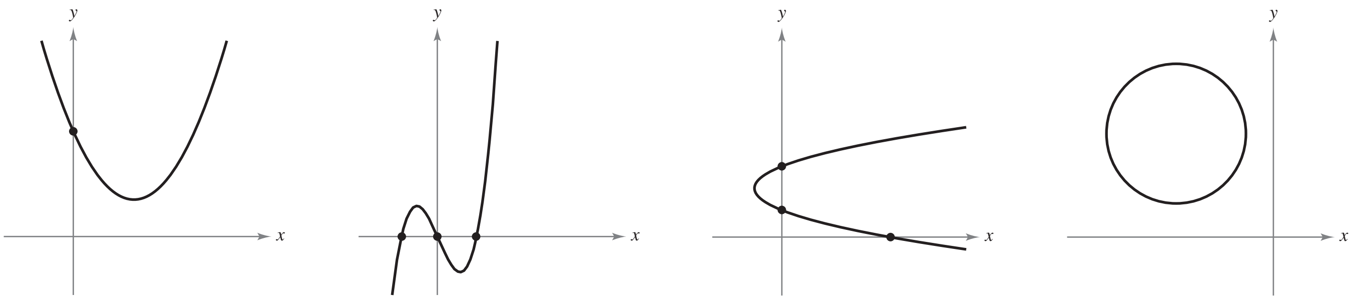
*In this text, the term *graphing utility* means either a graphing calculator, such as the *TI-Nspire*, or computer graphing software, such as *Maple* or *Mathematica*.

Intercepts of a Graph

REMARK Some texts denote the x-intercept as the x-coordinate of the point $(a, 0)$ rather than the point itself. Unless it is necessary to make a distinction, when the term *intercept* is used in this text, it will mean either the point or the coordinate.

Two types of solution points that are especially useful in graphing an equation are those having zero as their x- or y-coordinate. Such points are called **intercepts** because they are the points at which the graph intersects the x- or y-axis. The point $(a, 0)$ is an ***x*-intercept** of the graph of an equation when it is a solution point of the equation. To find the x-intercepts of a graph, let y be zero and solve the equation for x. The point $(0, b)$ is a ***y*-intercept** of the graph of an equation when it is a solution point of the equation. To find the y-intercepts of a graph, let x be zero and solve the equation for y.

It is possible for a graph to have no intercepts, or it might have several. For instance, consider the four graphs shown in Figure P.5.



No x-intercepts
One y-intercept

Three x-intercepts
One y-intercept

One x-intercept
Two y-intercepts

No intercepts

Figure P.5

EXAMPLE 2 Finding x- and y-Intercepts

Find the x- and y-intercepts of the graph of $y = x^3 - 4x$.

Solution To find the x-intercepts, let y be zero and solve for x.

$$x^3 - 4x = 0 \qquad \text{Let } y \text{ be zero.}$$

$$x(x - 2)(x + 2) = 0 \qquad \text{Factor.}$$

$$x = 0, 2, \text{ or } -2 \qquad \text{Solve for } x.$$

Because this equation has three solutions, you can conclude that the graph has three x-intercepts:

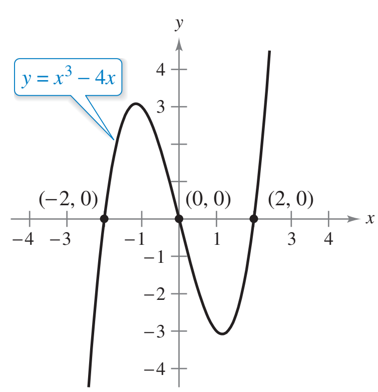
$$(0, 0), \quad (2, 0), \quad \text{and} \quad (-2, 0). \qquad x\text{-intercepts}$$

To find the y-intercepts, let x be zero. Doing this produces $y = 0$. So, the y-intercept is

$$(0, 0). \qquad y\text{-intercept}$$

(See Figure P.6.)

TECHNOLOGY Example 2 uses an analytic approach to finding intercepts. When an analytic approach is not possible, you can use a graphical approach by finding the points at which the graph intersects the axes. Use the *trace* feature of a graphing utility to approximate the intercepts of the graph of the equation in Example 2. Note that your utility may have a built-in program that can find the x-intercepts of a graph. (Your utility may call this the *root* or *zero* feature.) If so, use the program to find the x-intercepts of the graph of the equation in Example 2.



Intercepts of a graph
Figure P.6

Symmetry of a Graph

Knowing the symmetry of a graph before attempting to sketch it is useful because you need only half as many points to sketch the graph. The three types of symmetry listed below can be used to help sketch the graphs of equations (see Figure P.7).

1. A graph is **symmetric with respect to the *y*-axis** if, whenever (x, y) is a point on the graph, then $(-x, y)$ is also a point on the graph. This means that the portion of the graph to the left of the y-axis is a mirror image of the portion to the right of the y-axis.
2. A graph is **symmetric with respect to the *x*-axis** if, whenever (x, y) is a point on the graph, then $(x, -y)$ is also a point on the graph. This means that the portion of the graph below the x-axis is a mirror image of the portion above the x-axis.
3. A graph is **symmetric with respect to the origin** if, whenever (x, y) is a point on the graph, then $(-x, -y)$ is also a point on the graph. This means that the graph is unchanged by a rotation of 180° about the origin.

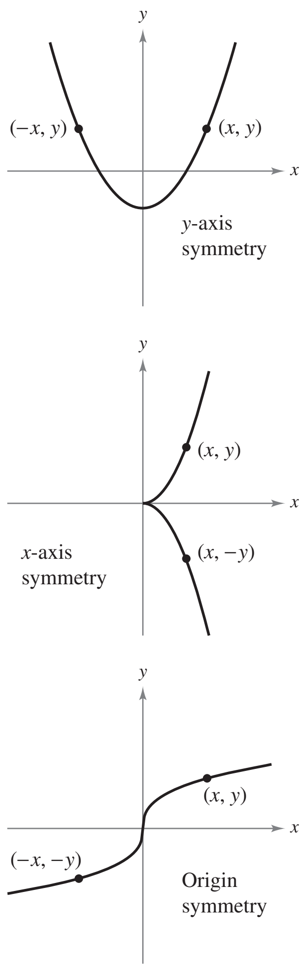


Figure P.7

Tests for Symmetry

1. The graph of an equation in x and y is symmetric with respect to the y-axis when replacing x by $-x$ yields an equivalent equation.
2. The graph of an equation in x and y is symmetric with respect to the x-axis when replacing y by $-y$ yields an equivalent equation.
3. The graph of an equation in x and y is symmetric with respect to the origin when replacing x by $-x$ and y by $-y$ yields an equivalent equation.

The graph of a polynomial has symmetry with respect to the y-axis when each term has an even exponent (or is a constant). For instance, the graph of

$$y = 2x^4 - x^2 + 2$$

has symmetry with respect to the y-axis. Similarly, the graph of a polynomial has symmetry with respect to the origin when each term has an odd exponent, as illustrated in Example 3.

EXAMPLE 3 Testing for Symmetry

Test the graph of $y = 2x^3 - x$ for symmetry with respect to (a) the y-axis and (b) the origin.

Solution

a. $y = 2x^3 - x$ Write original equation.

$y = 2(-x)^3 - (-x)$ Replace x by $-x$.

$y = -2x^3 + x$ Simplify. It is not an equivalent equation.

Because replacing x by $-x$ does *not* yield an equivalent equation, you can conclude that the graph of $y = 2x^3 - x$ is *not* symmetric with respect to the y-axis.

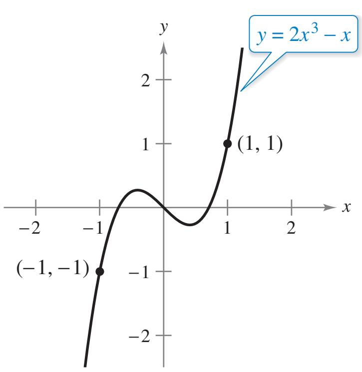
b. $y = 2x^3 - x$ Write original equation.

$-y = 2(-x)^3 - (-x)$ Replace x by $-x$ and y by $-y$.

$-y = -2x^3 + x$ Simplify.

$y = 2x^3 - x$ Equivalent equation

Because replacing x by $-x$ and y by $-y$ yields an equivalent equation, you can conclude that the graph of $y = 2x^3 - x$ is symmetric with respect to the origin, as shown in Figure P.8.



Origin symmetry
Figure P.8

EXAMPLE 4 Using Intercepts and Symmetry to Sketch a Graph

See LarsonCalculus.com for an interactive version of this type of example.

Sketch the graph of $x - y^2 = 1$.

Solution The graph is symmetric with respect to the x-axis because replacing y by $-y$ yields an equivalent equation.

$x - y^2 = 1$	Write original equation.
$x - (-y)^2 = 1$	Replace y by $-y$.
$x - y^2 = 1$	Equivalent equation

This means that the portion of the graph below the x-axis is a mirror image of the portion above the x-axis. To sketch the graph, first plot the x-intercept and the points above the x-axis. Then reflect in the x-axis to obtain the entire graph, as shown in Figure P.9.

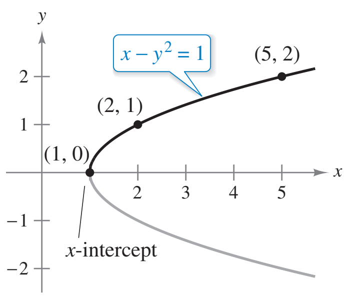


Figure P.9

TECHNOLOGY Graphing utilities are designed so that they most easily graph equations in which y is a function of x (see Section P.3 for a definition of **function**). To graph other types of equations, you need to split the graph into two or more parts *or* you need to use a different graphing mode. For instance, to graph the equation in Example 4, you can split it into two parts.

$y_1 = \sqrt{x - 1}$	Top portion of graph
$y_2 = -\sqrt{x - 1}$	Bottom portion of graph

Points of Intersection

A **point of intersection** of the graphs of two equations is a point that satisfies both equations. You can find the point(s) of intersection of two graphs by solving their equations simultaneously.

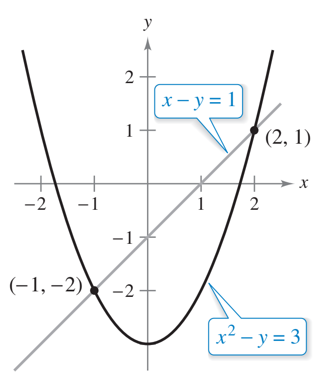
EXAMPLE 5 Finding Points of Intersection

Find all points of intersection of the graphs of

$$x^2 - y = 3 \quad \text{and} \quad x - y = 1.$$

Solution Begin by sketching the graphs of both equations in the *same* rectangular coordinate system, as shown in Figure P.10. From the figure, it appears that the graphs have two points of intersection. You can find these two points as follows.

$y = x^2 - 3$	Solve first equation for y.
$y = x - 1$	Solve second equation for y.
$x^2 - 3 = x - 1$	Equate y-values.
$x^2 - x - 2 = 0$	Write in general form.
$(x - 2)(x + 1) = 0$	Factor.
$x = 2 \text{ or } -1$	Solve for x.



Two points of intersection
Figure P.10

The corresponding values of y are obtained by substituting $x = 2$ and $x = -1$ into either of the original equations. Doing this produces two points of intersection:

$(2, 1)$ and $(-1, -2)$. Points of intersection

You can check the points of intersection in Example 5 by substituting into *both* of the original equations or by using the *intersect* feature of a graphing utility.

Mathematical Models

Real-life applications of mathematics often use equations as **mathematical models.** In developing a mathematical model to represent actual data, you should strive for two (often conflicting) goals: accuracy and simplicity. That is, you want the model to be simple enough to be workable, yet accurate enough to produce meaningful results. Section P.4 explores these goals more completely.

The Mauna Loa Observatory in Hawaii has been measuring the increasing concentration of carbon dioxide in Earth's atmosphere since 1958.

EXAMPLE 6 Comparing Two Mathematical Models

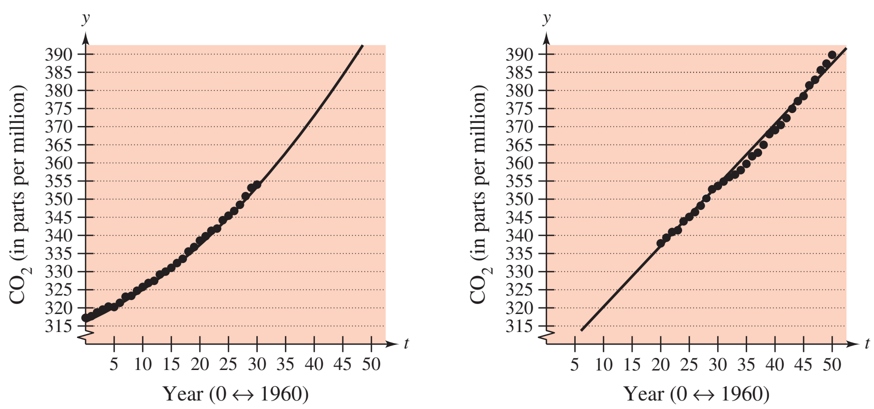
The Mauna Loa Observatory in Hawaii records the carbon dioxide concentration y (in parts per million) in Earth's atmosphere. The January readings for various years are shown in Figure P.11. In the July 1990 issue of *Scientific American*, these data were used to predict the carbon dioxide level in Earth's atmosphere in the year 2035, using the quadratic model

$$y = 0.018t^2 + 0.70t + 316.2$$ Quadratic model for 1960–1990 data

where $t = 0$ represents 1960, as shown in Figure P.11(a). The data shown in Figure P.11(b) represent the years 1980 through 2010 and can be modeled by

$$y = 1.68t + 303.5$$ Linear model for 1980–2010 data

where $t = 0$ represents 1960. What was the prediction given in the *Scientific American* article in 1990? Given the new data for 1990 through 2010, does this prediction for the year 2035 seem accurate?



(a) **(b)**

Figure P.11

Solution To answer the first question, substitute $t = 75$ (for 2035) into the quadratic model.

$$y = 0.018(75)^2 + 0.70(75) + 316.2 = 469.95$$ Quadratic model

So, the prediction in the *Scientific American* article was that the carbon dioxide concentration in Earth's atmosphere would reach about 470 parts per million in the year 2035. Using the linear model for the 1980–2010 data, the prediction for the year 2035 is

$$y = 1.68(75) + 303.5 = 429.5.$$ Linear model

So, based on the linear model for 1980–2010, it appears that the 1990 prediction was too high.

The models in Example 6 were developed using a procedure called *least squares regression* (see Section 13.9). The quadratic and linear models have correlations given by $r^2 \approx 0.997$ and $r^2 \approx 0.994$, respectively. The closer r^2 is to 1, the "better" the model.

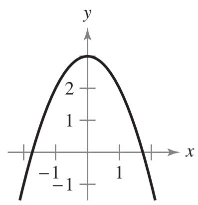
Gavriel Jecan/Terra/CORBIS

P.1 Exercises

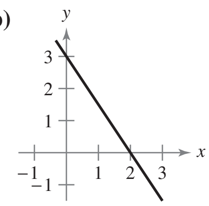
See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Matching **In Exercises 1–4, match the equation with its graph. [The graphs are labeled (a), (b), (c), and (d).]**

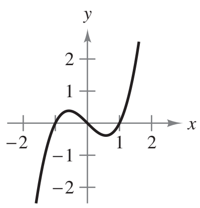
(a)



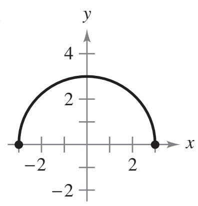
(b)



(c)



(d)



1. $y = -\frac{3}{2}x + 3$

2. $y = \sqrt{9 - x^2}$

3. $y = 3 - x^2$

4. $y = x^3 - x$

Sketching a Graph by Point Plotting **In Exercises 5–14, sketch the graph of the equation by point plotting.**

5. $y = \frac{1}{2}x + 2$

6. $y = 5 - 2x$

7. $y = 4 - x^2$

8. $y = (x - 3)^2$

9. $y = |x + 2|$

10. $y = |x| - 1$

11. $y = \sqrt{x} - 6$

12. $y = \sqrt{x + 2}$

13. $y = \dfrac{3}{x}$

14. $y = \dfrac{1}{x + 2}$

Approximating Solution Points **In Exercises 15 and 16, use a graphing utility to graph the equation. Move the cursor along the curve to approximate the unknown coordinate of each solution point accurate to two decimal places.**

15. $y = \sqrt{5 - x}$
(a) $(2, y)$
(b) $(x, 3)$

16. $y = x^5 - 5x$
(a) $(-0.5, y)$
(b) $(x, -4)$

Finding Intercepts **In Exercises 17–26, find any intercepts.**

17. $y = 2x - 5$

18. $y = 4x^2 + 3$

19. $y = x^2 + x - 2$

20. $y^2 = x^3 - 4x$

21. $y = x\sqrt{16 - x^2}$

22. $y = (x - 1)\sqrt{x^2 + 1}$

23. $y = \dfrac{2 - \sqrt{x}}{5x + 1}$

24. $y = \dfrac{x^2 + 3x}{(3x + 1)^2}$

25. $x^2y - x^2 + 4y = 0$

26. $y = 2x - \sqrt{x^2 + 1}$

Testing for Symmetry **In Exercises 27–38, test for symmetry with respect to each axis and to the origin.**

27. $y = x^2 - 6$

28. $y = x^2 - x$

29. $y^2 = x^3 - 8x$

30. $y = x^3 + x$

31. $xy = 4$

32. $xy^2 = -10$

33. $y = 4 - \sqrt{x + 3}$

34. $xy - \sqrt{4 - x^2} = 0$

35. $y = \dfrac{x}{x^2 + 1}$

36. $y = \dfrac{x^2}{x^2 + 1}$

37. $y = |x^3 + x|$

38. $|y| - x = 3$

Using Intercepts and Symmetry to Sketch a Graph **In Exercises 39–56, find any intercepts and test for symmetry. Then sketch the graph of the equation.**

39. $y = 2 - 3x$

40. $y = \frac{2}{3}x + 1$

41. $y = 9 - x^2$

42. $y = 2x^2 + x$

43. $y = x^3 + 2$

44. $y = x^3 - 4x$

45. $y = x\sqrt{x + 5}$

46. $y = \sqrt{25 - x^2}$

47. $x = y^3$

48. $x = y^2 - 4$

49. $y = \dfrac{8}{x}$

50. $y = \dfrac{10}{x^2 + 1}$

51. $y = 6 - |x|$

52. $y = |6 - x|$

53. $y^2 - x = 9$

54. $x^2 + 4y^2 = 4$

55. $x + 3y^2 = 6$

56. $3x - 4y^2 = 8$

Finding Points of Intersection **In Exercises 57–62, find the points of intersection of the graphs of the equations.**

57. $x + y = 8$
$4x - y = 7$

58. $3x - 2y = -4$
$4x + 2y = -10$

59. $x^2 + y = 6$
$x + y = 4$

60. $x = 3 - y^2$
$y = x - 1$

61. $x^2 + y^2 = 5$
$x - y = 1$

62. $x^2 + y^2 = 25$
$-3x + y = 15$

Finding Points of Intersection **In Exercises 63–66, use a graphing utility to find the points of intersection of the graphs. Check your results analytically.**

63. $y = x^3 - 2x^2 + x - 1$
$y = -x^2 + 3x - 1$

64. $y = x^4 - 2x^2 + 1$
$y = 1 - x^2$

65. $y = \sqrt{x + 6}$
$y = \sqrt{-x^2 - 4x}$

66. $y = -|2x - 3| + 6$
$y = 6 - x$

The symbol indicates an exercise in which you are instructed to use graphing technology or a symbolic computer algebra system. The solutions of other exercises may also be facilitated by the use of appropriate technology.

67. Modeling Data The table shows the Gross Domestic Product, or GDP (in trillions of dollars), for selected years. *(Source: U.S. Bureau of Economic Analysis)*

Year	1980	1985	1990	1995
GDP	2.8	4.2	5.8	7.4

Year	2000	2005	2010
GDP	10.0	12.6	14.5

(a) Use the regression capabilities of a graphing utility to find a mathematical model of the form $y = at^2 + bt + c$ for the data. In the model, y represents the GDP (in trillions of dollars) and t represents the year, with $t = 0$ corresponding to 1980.

(b) Use a graphing utility to plot the data and graph the model. Compare the data with the model.

(c) Use the model to predict the GDP in the year 2020.

68. Modeling Data

The table shows the numbers of cellular phone subscribers (in millions) in the United States for selected years. *(Source: CTIA-The Wireless)*

Year	1995	1998	2001	2004	2007	2010
Number	34	69	128	182	255	303

(a) Use the regression capabilities of a graphing utility to find a mathematical model of the form $y = at^2 + bt + c$ for the data. In the model, y represents the number of subscribers (in millions) and t represents the year, with $t = 5$ corresponding to 1995.

(b) Use a graphing utility to plot the data and graph the model. Compare the data with the model.

(c) Use the model to predict the number of cellular phone subscribers in the United States in the year 2020.

69. Break-Even Point Find the sales necessary to break even $(R = C)$ when the cost C of producing x units is $C = 2.04x + 5600$ and the revenue R from selling x units is $R = 3.29x$.

70. Copper Wire The resistance y in ohms of 1000 feet of solid copper wire at 77°F can be approximated by the model

$$y = \frac{10{,}770}{x^2} - 0.37, \quad 5 \le x \le 100$$

where x is the diameter of the wire in mils (0.001 in.). Use a graphing utility to graph the model. By about what factor is the resistance changed when the diameter of the wire is doubled?

71. Using Solution Points For what values of k does the graph of $y = kx^3$ pass through the point?

(a) $(1, 4)$ (b) $(-2, 1)$ (c) $(0, 0)$ (d) $(-1, -1)$

72. Using Solution Points For what values of k does the graph of $y^2 = 4kx$ pass through the point?

(a) $(1, 1)$ (b) $(2, 4)$ (c) $(0, 0)$ (d) $(3, 3)$

WRITING ABOUT CONCEPTS

Writing Equations In Exercises 73 and 74, write an equation whose graph has the indicated property. (There may be more than one correct answer.)

73. The graph has intercepts at $x = -4$, $x = 3$, and $x = 8$.

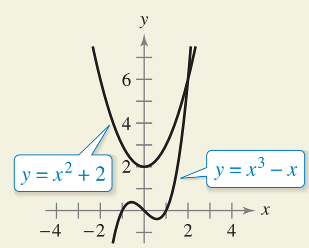
74. The graph has intercepts at $x = -\frac{3}{2}$, $x = 4$, and $x = \frac{5}{2}$.

75. Proof

(a) Prove that if a graph is symmetric with respect to the x-axis and to the y-axis, then it is symmetric with respect to the origin. Give an example to show that the converse is not true.

(b) Prove that if a graph is symmetric with respect to one axis and to the origin, then it is symmetric with respect to the other axis.

76. HOW DO YOU SEE IT? Use the graphs of the two equations to answer the questions below.



(a) What are the intercepts for each equation?

(b) Determine the symmetry for each equation.

(c) Determine the point of intersection of the two equations.

True or False? In Exercises 77–80, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

77. If $(-4, -5)$ is a point on a graph that is symmetric with respect to the x-axis, then $(4, -5)$ is also a point on the graph.

78. If $(-4, -5)$ is a point on a graph that is symmetric with respect to the y-axis, then $(4, -5)$ is also a point on the graph.

79. If $b^2 - 4ac > 0$ and $a \ne 0$, then the graph of $y = ax^2 + bx + c$ has two x-intercepts.

80. If $b^2 - 4ac = 0$ and $a \ne 0$, then the graph of $y = ax^2 + bx + c$ has only one x-intercept.

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